EXAM 2

19 October 1999

IMPORTANT: Write clearly and neatly. Make sure that you give some reasoning or working for each answer. Full marks will NOT be awarded for the final answer by itself, UNLESS it is supported by a brief justification or explanation.
Give units for all numerical quantities!

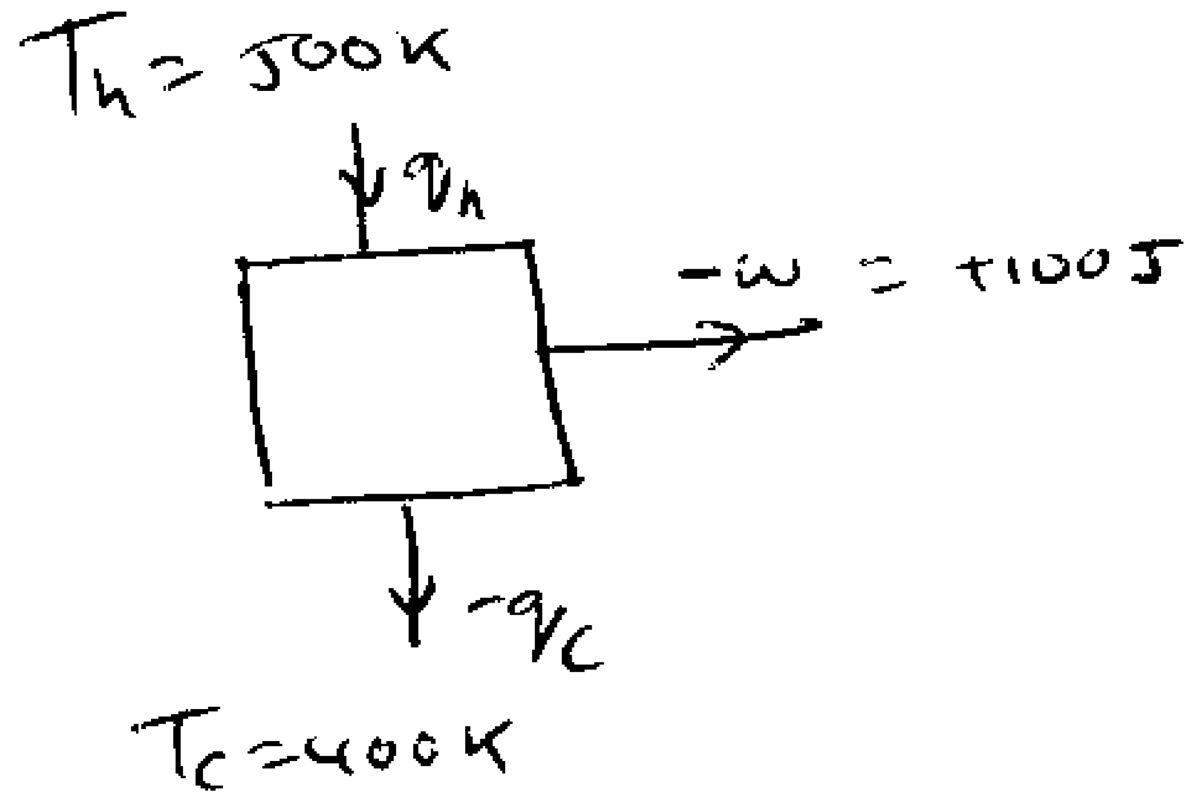
Some data: $R = 8.314\ J\ K^{-1}\ mol^{-1}$ $1\ atm = 101325\ Pa$ $N_A = 6.022 \times 10^{23}\ mol^{-1}$
$k = 1.38 \times 10^{-23}\ J\ K^{-1}\ mol^{-1}$ $1\ amu = 1.66 \times 10^{-27}\ kg$

Your name SOLUTIONS

(1) *30 points*

a) An ideal Carnot engine operates between 500 K and 400 K. If 100 J of work is obtained from this engine, how much heat is absorbed at the higher temperature and how much is rejected at the lower?
b) Prove that for the reversible isothermal expansion of 1 mol of ideal gas from V_1 to V_2, $\Delta S = R \ln (V_2/V_1)$. HINT: $dU = 0$ for an isothermal process involving an ideal gas.

a)



$$\varepsilon = \frac{T_h - T_c}{T_h} = 0.2 = \frac{-w}{q_h}$$

$$\therefore q_h = 500\ J \quad \therefore q_c = -400\ J.$$

b) $$0 = dU = dq + dw = dq - p\,dV = dq - \frac{RT}{V}dV$$

$$dS = \frac{dq}{T} = \frac{R\,dV}{V} \quad \therefore \quad \Delta S = \int_{V_1}^{V_2} R\frac{dV}{V} = R \ln\left(\frac{V_2}{V_1}\right)$$

(2) *30 points*

1 mol of an imaginary gas has $U = T\,V^2$.

Find $(\partial U/\partial V)_T$, C_V and $(\partial C_V/\partial V)_T$.

$$\left(\frac{\partial U}{\partial V}\right)_T = 2TV$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = V^2$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = 2V$$

(3) *40 points*

Two identical metal blocks (together they form the system), 1 and 2, each have a heat capacity at constant pressure of C_p and are initially at temperatures T_1 and T_2, where $T_1 < T_2$. C_p varies with temperature and is given by $a + bT$ where a and b are constants. The system is maintained at constant pressure.

a) Heat is transferred reversibly from one to the other until they are both at a final temperature of $(T_1 + T_2)/2$. What is ΔS for the system, surroundings and universe?

b) Instead the heat transfer was accomplished spontaneously, by touching the blocks together. In this second case, what is ΔS for the system, surroundings and universe?

a) $dS = \frac{dq_{rev}}{T} = \frac{C_p\,dT}{T}$.

ΔS_1 for block 1 $= \int_{T_1}^{(T_1+T_2)/2} \frac{C_p\,dT}{T} = \left[a \ln T + bT\right]_{T_1}^{\frac{T_1+T_2}{2}}$

ΔS_2 for block 2 $= \left[a \ln T + bT\right]_{T_2}^{(T_1+T_2)/2}$

$\Delta S_{sys} = \Delta S_1 + \Delta S_2 = a \ln\left(\frac{(T_1+T_2)^2}{4T_1T_2}\right)$.

Reversible so $\Delta S_{surr} = -\Delta S_{sys}$ and $\Delta S_{uni} = 0$.

b) ΔS_{sys} the same (state function).

$\Delta S_{surr} = 0$ because no heat transfer to surroundings.

$\Delta S_{uni} = \Delta S_{sys}$ here.